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Higgs mass bounds in a Triplet Model

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Abstract

We perform a global fit to high energy precision electroweak data in a Higgs model containing the usual isospin doublet plus a real isospin triplet. The analysis is performed in terms of the oblique parameters S, T and U and we show that the mass of the lightest Higgs boson can be as large as 2 TeV.

1 Introduction

With the high-energy measurements of electroweak (EW) observables by LEP and SLD [1], an impressive level of precision has been achieved, in many cases to 0.1%. These have confirmed the Glashow-Salam-Weinberg model of EW broken gauge symmetry with great certainty. What remains is to discover the nature of the symmetry breaking. If it is the Standard Model (SM) Higgs, i.e. a complex isospin doublet, the hard empirical lower bound we have on its mass is the current 113 GeV from the LEP search [1].

The other empirical bound on the SM Higgs is less direct. It involves constraining the radiative corrections to EW observables already measured. One feature of the virtual Higgs corrections is that, to a very good approximation, they are *oblique*, i.e. they appear only in the corrections to propagators of the EW gauge bosons [2]. It happens that the important effects can be summarised by two parameters, S and T [3], which on the one hand can be calculated in the SM, and on the other hand fitted globally to all current precision data. Another important feature is that the dependence of S and T on the SM Higgs mass m_h is logarithmic. However, although the m_h-dependence is weak, one can nevertheless put an upper limit $m_h < 165$ GeV at the 95% confidence level [1].

Such an upper limit on m_h is necessarily model-dependent, in the sense that it applies only to the minimal SM scenario. In this letter, we consider a Higgs model [4, 5] (TM) with the SM complex doublet plus a real triplet of scalars. The physical spectrum contains two extra states, namely another neutral k^0 and a charged $h^\pm$. The model violates the custodial symmetry responsible for the tree-level relation

$$\rho \equiv \frac{m_W^2}{m_Z^2 c_W^2} = 1, \tag{1}$$

where $c_W = \cos\theta_W = g/\sqrt{(g^2 + g'^2)}$ but by making the triplet vev small the relation can be satisfied to within the experimental uncertainties.

In this small vev approximation, we shall show that the tree level corrections can be absorbed into a shift in T which is of the correct sign to partially cancel the SM Higgs contribution. We also compute the one-loop corrections where the two new particles contribute directly to the oblique parameters: S and U are largely unaffected whilst T receives a correction depending on the ratio $(\Delta m/m_Z)^2$, where Δm is the mass splitting between the k^0 and $h^\pm$. Like the tree-level correction, this is also of the correct sign to partially cancel the SM Higgs contribution. We demonstrate that it is consequently possible to relax the upper limit on m_h to values as large as 2 TeV.

The discussion below is arranged as follows: Section 2 gives the Lagrangian for the triplet model (TM) and a brief description of its spectrum; in Section 3 we define the parameters S, T and U then show how any given EW observable depends on them, and give the result of their calculation in the SM and the TM; in Section 4 we compare the SM and TM calculations with our fit to the EW data. Finally, we make some concluding remarks.

2 The Model

The Lagrangian for the model, containing one complex Higgs doublet and one real triplet is

$$\begin{aligned}\mathcal{L}(\Phi, H) &= (D_\mu\Phi)^\dagger(D^\mu\Phi) + \tfrac{1}{2}(D_\mu H)^\dagger(D^\mu H) - V(\Phi, H), \\ V(\Phi, H) &= \mu_1^2\,\Phi^\dagger\Phi + \tfrac{1}{2}\mu_2^2\,H^\dagger H \\ &+ \lambda_1\,(\Phi^\dagger\Phi)^2 + \tfrac{1}{4}\lambda_2\,(H^\dagger H)^2 \\ &+ \tfrac{1}{2}\lambda_3\,(\Phi^\dagger\Phi)(H^\dagger H) + \lambda_4\,v\,H_U^i\,\Phi^\dagger\sigma^i\Phi. \end{aligned} \tag{2}$$

The field components are, including the neutral components' vevs,

$$\Phi = \begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}}(v + \phi_R^0 + i\phi_I^0) \end{pmatrix}, \qquad H = \begin{pmatrix} \eta^+ \\ \frac{1}{2}vt_\beta + \eta^0 \\ -\eta^- \end{pmatrix}. \tag{3}$$

In the above, σ^i are the Pauli matrices and $t_\beta = \tan\beta$. H is even under charge conjugation, i.e.

$$H = H_c = C\,H^*, \tag{4}$$

where

$$C = \begin{pmatrix} 0 & 0 & -1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{pmatrix}. \tag{5}$$

and can be cast into a form, H_U, involving only real fields, by the unitary transformation

$$H_U = U^\dagger H. \tag{6}$$

where

$$U = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & -i & 0 \\ 0 & 0 & \sqrt{2} \\ -1 & -i & 0 \end{pmatrix}. \tag{7}$$

Expanding about the vacuum by substituting eq. (3) into the Lagrangian, we can analyse the mass spectrum. One finds two charged Higgs states. The first, $g^\pm$, is massless and is the Goldstone to be eaten by the $W^\pm$. The second we call $h^\pm$, having mass m_c such that

$$m_c^2 = \frac{\lambda_4 v^2}{s_\beta c_\beta} \sim \frac{\lambda_4 v^2}{\beta}. \tag{8}$$

In terms of the original doublet and triplet charged components these are

$$\begin{pmatrix} g^\pm \\ h^\pm \end{pmatrix} = \begin{pmatrix} c_\beta & s_\beta \\ -s_\beta & c_\beta \end{pmatrix}\begin{pmatrix} \phi^\pm \\ \eta^\pm \end{pmatrix}. \tag{9}$$

In the charge neutral sector we have a CP-odd massless state which is the Goldstone g^0 to be eaten by the Z^0:

$$g^0 = \phi_I^0. \tag{10}$$

Finally, there are two CP-even states, called h^0 and k^0, having mass m_h and m_k respectively. In terms of the original doublet and triplet components there is generically a mixing:

$$\begin{pmatrix} h^0 \\ k^0 \end{pmatrix} = \begin{pmatrix} c_\gamma & s_\gamma \\ -s_\gamma & c_\gamma \end{pmatrix} \begin{pmatrix} \phi^0 \\ \eta^0 \end{pmatrix}. \tag{11}$$

For simplicity, we shall only consider the case of zero mixing, $\gamma = 0$, leading to masses

$$m_h^2 = 2\,\lambda_1\, v^2, \tag{12}$$

$$m_k^2 = \tfrac{1}{2}\,\lambda_2\,(v t_\beta)^2 + \frac{\lambda_4 v^2}{t_\beta}. \tag{13}$$

The model has six parameters, $\mu_{1,2}$ and $\lambda_{1,2,3,4}$, or alternatively v, β, m_c, m_h, m_k and γ. However, since we assume zero mixing between the neutral CP even Higgses this reduces to five parameters because $\lambda_3 = 2\lambda_4/t_\beta$.

The most important tree-level predictions of the model are the masses for the $W^\pm$ and Z^0, which are

$$m_W = \frac{gv}{2c_\beta}, \qquad m_Z = \frac{gv}{2c_W}, \tag{14}$$

where $c_\beta = \cos\beta$. The expression for m_Z is identical to the SM with just the doublet, while m_W is increased relative to the SM. This gives a tree-level ρ-parameter:

$$\rho \equiv \frac{m_W^2}{m_Z^2 c_W^2} = \frac{1}{c_\beta^2}. \tag{15}$$

Thus s_β has to be less than a few percent in order to have a realistic phenomenology.

An important issue is that taking $s_\beta \to 0$, keeping all other parameters in the Lagrangian fixed, is a decoupling limit such that all effects of the extra Higgs triplet on EW observables become negligible. However, and as we shall soon show, it is quite possible to have interesting phenomenology with small but non-zero β.

3 Oblique Corrections

The parameters S, T and U are defined as

$$\begin{aligned} \alpha\, S &= \frac{4 s_W^2 c_W^2}{m_Z^2} \left(\Delta\Pi^{ZZ}(m_Z) - \frac{c_W^2 - s_W^2}{s_W c_W}\, \Delta\Pi^{\gamma Z}(m_Z) - \Delta\Pi^{\gamma\gamma}(m_Z) \right), \\ \alpha\, T &= \frac{1}{m_W^2} \left(\Pi^{WW}(0) - c_W^2\, \Pi^{ZZ}(0) \right), \\ \alpha\,(S+U) &= 4 s_W^2 \left(\frac{\Delta\Pi^{WW}(m_W)}{m_W^2} - \frac{c_W}{s_W} \frac{\Delta\Pi^{\gamma Z}(m_Z)}{m_Z^2} - \frac{\Delta\Pi^{\gamma\gamma}(m_Z)}{m_Z^2} \right), \end{aligned} \tag{16}$$

where $\Delta\Pi(k) = \Pi(k) - \Pi(0)$. The functions $\Pi(k)$ are the coefficients of the metric in the one-loop gauge boson inverse propagators:

$$\Pi_{\mu\nu}(k) = g_{\mu\nu}\, \Pi(k). \tag{17}$$

Predictions for EW observables, which we write generically as $\mathcal{O}$, can be written in terms of S, T and U. If we are just considering the prediction of the SM, we can write

$$
\begin{aligned}
\mathcal{O}_{SM}(m_h) &= \mathcal{O}_{SM}(m_h^{ref}) \\
&+ \alpha\, A_{SM}\, \Delta S_{SM}(m_h, m_h^{ref}) \\
&+ \alpha\, B_{SM}\, \Delta T_{SM}(m_h, m_h^{ref}) \\
&+ \alpha\, C_{SM}\, \Delta U_{SM}(m_h, m_h^{ref}). \qquad (18)
\end{aligned}
$$

Here, $\mathcal{O}_{SM}$ is the one-loop SM prediction for the observable, in terms of the input parameters $\alpha(0)$, m_Z, G_μ, m_t, $\alpha_s(m_Z)$ and $\Delta\alpha_{had}^{(5)}(m_Z)$. The first term on the r.h.s. is the SM prediction evaluated at a fixed reference Higgs mass, which is arbitrary. The coefficients A_{SM}, B_{SM} and C_{SM} are process dependent but independent of the new physics (which in this case is that of the Higgs). ΔS_{SM}, ΔT_{SM} and ΔU_{SM} are the contributions to S, T and U after subtracting their value at the reference Higgs mass, i.e. they are defined to vanish when $m_h = m_h^{ref}$. In this way one can quantify the effect of varying the Higgs mass on the observable simply in terms of ΔS_{SM}, ΔT_{SM} and ΔU_{SM}.

If now we consider the prediction of the TM, some modifications are required. In this case a general observable (setting $\gamma = 0$) is written as follows:

$$
\begin{aligned}
\mathcal{O}_{TM}(m_h, m_k, m_c, \beta) &= \mathcal{O}_{SM}(m_h^{ref}) \\
&+ A_{SM}\,(\alpha\ \Delta S_{SM}(m_h, m_h^{ref})\ +\ \alpha\ S_{TM}(m_k, m_c)) \\
&+ B_{SM}\,(\alpha\ \Delta T_{SM}(m_h, m_h^{ref})\ +\ \alpha\ T_{TM}(m_k, m_c) + \delta_{tree}(\beta)), \\
&+ C_{SM}\,(\alpha\ \Delta U_{SM}(m_h, m_h^{ref})\ +\ \alpha\ U_{TM}(m_k, m_c)). \qquad (19)
\end{aligned}
$$

Here we have extra contributions, denoted with TM in subscript, coming from the extra k^0 and $h^\pm$ loops. Since we are taking β^2 to be small, there are some simplifications. To the accuracy we require, $O(\alpha\beta^2)$ may be neglected, as may $O(\beta^4)$. Therefore, we may evaluate our one-loop corrections, themselves of $O(\alpha)$, at $\beta = 0$, so that the coefficients of S_{TM} , T_{TM} and U_{TM} are the same as in the SM. The only β-dependence takes the form of $O(\beta^2)$ corrections that occur at tree-level and these are contained in the correction $\delta_{tree}(\beta)$. There appear to be two distinct types of contribution to $\delta_{tree}(\beta)$:

1. Direct tree-level corrections. In our case, only one observable, m_W, has a direct tree-level correction, as seen in eq.(15). This is because it is the only high-energy EW observable we shall fit to which involves the W boson at tree level.

2. Indirect tree-level corrections: All the EW observables (except m_W as we have just mentioned) can be written at tree-level in terms of α, m_Z and s_W, none of which depends directly on β in the TM. However, s_W is constrained using the input datum G_μ which does itself have a dependence on β. At tree level, this is

$$\sqrt{2}\, G_\mu \;=\; \frac{g^2}{4m_W^2} \;=\; \frac{4\pi\alpha}{m_Z^2}\,\frac{c_\beta^2}{\sin^2 2\theta_W}. \qquad (20)$$

All observables we consider receive an indirect shift whilst only the W mass picks up a direct shift. However, since the shift is essentially oblique it can in all cases be absorbed into a shift in T, as we anticipated in eq.(19). In particular it is straightforward to show that, to leading order,

$$\delta_{tree} = \beta^2. \tag{21}$$

For use later we define

$$\begin{aligned} S &= \Delta S_{SM} + S_{TM} \\ \alpha\, T &= \alpha\, \Delta T_{SM} + \alpha\, T_{TM} + \delta_{tree} \\ U &= \Delta U_{SM} + U_{TM} \end{aligned} \tag{22}$$

Calculation of the SM Higgs boson contributions to S and T are as follows (evaluated at m_Z):

$$\begin{aligned} S_{\rm SM} &= \frac{1}{\pi}\left[\frac{3}{8}\frac{m_h^2}{m_Z^2} - \frac{1}{12}\frac{m_h^4}{m_Z^4}\right. \\ &+ \frac{m_h^2}{m_Z^2}\log\left(\frac{m_h^2}{m_Z^2}\right)\left(\frac{3m_Z^2 - m_h^2}{4m_Z^2} + \frac{1}{24}\frac{m_h^4}{m_Z^4} + \frac{3m_Z^2}{4(m_Z^2 - m_h^2)}\right) \\ &+ \left(1 - \frac{1}{3}\frac{m_h^2}{m_Z^2} + \frac{1}{12}\frac{m_h^4}{m_Z^4}\right)\frac{m_h}{m_Z^2} \\ &\times \left.\left\{\begin{array}{ll} \sqrt{4m_Z^2 - m_h^2}\,\tan^{-1}\sqrt{\frac{4m_Z^2 - m_h^2}{m_h^2}}\,; & m_h < 2m_Z \\ \sqrt{m_h^2 - 4m_Z^2}\,\log\left(\frac{2m_Z}{m_h + \sqrt{m_h^2 - 4m_Z^2}}\right); & m_h > 2m_Z \end{array}\right\}\right], \end{aligned} \tag{23}$$

$$T_{\rm SM} = \frac{3}{16\pi}\frac{1}{s_W^2 c_W^2}\left[\frac{m_h^2}{m_Z^2 - m_h^2}\log\left(\frac{m_h^2}{m_Z^2}\right) - \frac{c_W^2 m_h^2}{c_W^2 m_Z^2 - m_h^2}\log\left(\frac{m_h^2}{c_W^2 m_Z^2}\right)\right]. \tag{24}$$

We do not show U_{SM} since is depends very weakly on m_h.

The TM contributions, to leading order in β, are (see Appendix)

$$S_{TM} = 0, \tag{25}$$

$$\begin{aligned} T_{TM} &= \frac{1}{8\pi}\frac{1}{s_W^2 c_W^2}\left[\frac{m_k^2 + m_c^2}{m_Z^2} - \frac{2m_c^2 m_k^2}{m_Z^2(m_k^2 - m_c^2)}\log\left(\frac{m_k^2}{m_c^2}\right)\right], \\ &\simeq \frac{1}{6\pi}\frac{1}{s_W^2 c_W^2}\frac{(\Delta m)^2}{m_Z^2}. \\ U_{TM} &= -\frac{1}{3\pi}\left(m_k^4\log\left(\frac{m_k^2}{m_c^2}\right)\frac{(3m_c^2 - m_k^2)}{(m_k^2 - m_c^2)^3} + \frac{5(m_k^4 + m_c^4) - 22m_k^2 m_c^2}{6(m_k^2 - m_c^2)^2}\right) + O(m_Z/m_c) \\ &\simeq \frac{\Delta m}{3\pi m_c}. \end{aligned} \tag{26}$$

Notice that the TM contribution to S is zero to this order. The TM contribution to T is positive and, in the approximation of $\Delta m = m_k - m_c \ll m_c$, has the rough power dependence shown above. U also vanishes when $\Delta m \to 0$, and falls to zero at

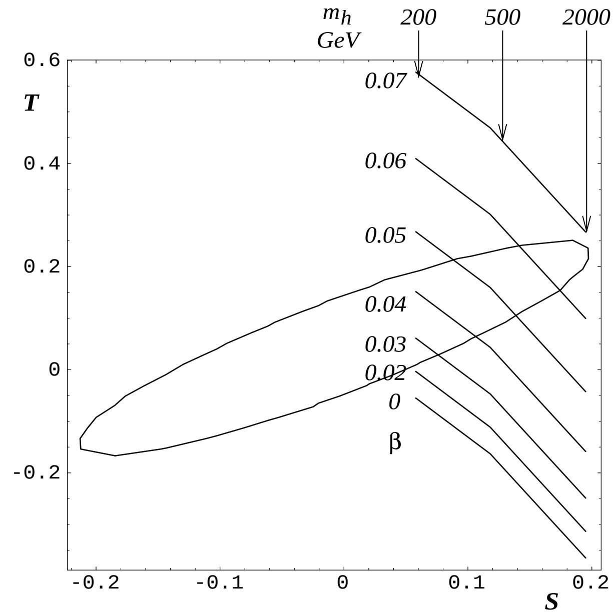

Figure 1: Ellipse encloses the region allowed by data. Curves show results in the TM for various values of β and various doublet Higgs masses. $\Delta m = 0$ and $U = 0$ in this plot.

large triplet masses. In particular, it has a negligible effect on all the results we shall subsequently show provided m_k, $m_c > 1$ TeV.

We have thus shown that the TM generates a positive correction to T due both to tree-level mixing and quantum loops. As we shall demonstrate in the next section, this allows us to compensate for an increase in the doublet Higgs mass thus releasing the SM upper bound.

We note that the quantum corrections are important for $\Delta m \sim m_Z$ and that this is possible provided $\lambda_2 \gg \lambda_4$, e.g. $\lambda_4 \sim \beta$, $\lambda_3 \sim 1$, $\lambda_2 \sim 1/\beta^2$ is a scenario which would lead to triplet bosons of mass $\sim v$. In such cases, λ_2 is large and the Higgs sector would become non-perturbative. More naturally, the triplet Higgs bosons are of mass $\sim v/\beta$ and the mass splitting is much less than m_Z. In this case, the principal contribution will arise from the tree-level mixing.

4 Comparison with Data

Using the program `ZFITTER` [6] we compute a total of 13 standard observables[1] with $m_h^{ref} = 100$ GeV, $m_t = 174.3$ GeV, $G_\mu = 1.6639 \times 10^{-5}$ GeV^2, $m_Z = 91.1875$ GeV, α_s =0.119 and $\Delta\alpha_{had}^{(5)}(m_Z)$= 0.02804. These results then determine the allowed region in $S-T$ parameter space. This is represented by the interior of the ellipse shown in Figures 2 and 1. The ellipse corresponds to a total chi-squared of 26.3 for the

[1] They are those listed in Table 41 of reference [1].

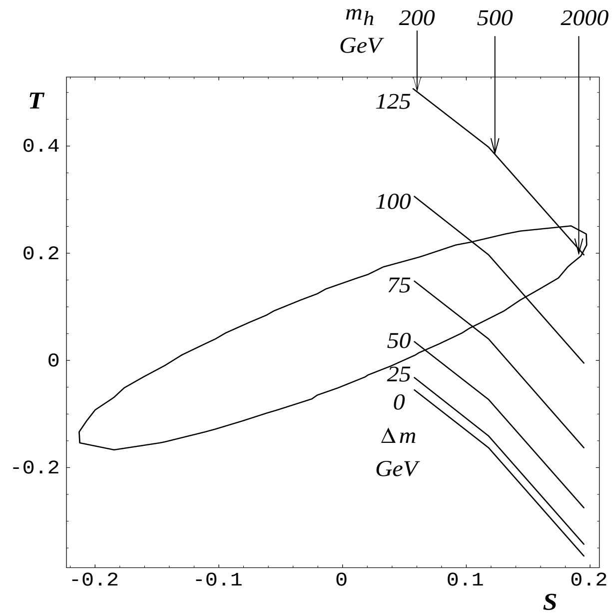

Figure 2: Ellipse encloses the region allowed by data. Curves show results in the TM for various mass splittings and various doublet Higgs masses. β and U are assumed to be negligibly small in this plot.

17 measurements used. We have investigated variations in the ellipse as the input parameters m_t, α_s and $\Delta\alpha^{(5)}_{had}$ are varied within their errors: a smaller value of $\alpha_s = 0.117$ is slightly favoured, varying m_t (± 5.1 GeV) leads to a shift ± 0.05 in T, whilst varying $\Delta\alpha^{(5)}_{had}$ (± 0.00065) leads to a shift of ± 0.05 in S.

In Figure 1 each line now shows the TM at a particular value of β for $\Delta m = 0$ (which turns off the quantum corrections) and m_h varying from 200 GeV to 2 TeV. We see that even in the absence of quantum corrections the TM is able to accommodate any m_h up to around 2 TeV and the mixing angle β cannot be much larger than 0.07.

In Figure 2 each line shows the TM result as m_h is varied, as before, at fixed Δm. β is assumed to be negligibly small in this plot (which turns off the tree-level correction δ_{tree}) and as a result the $\Delta m = 0$ line is identical to that which would arise in the SM. Clearly the quantum corrections contribute to T so as to allow any m_h up to around 2 TeV and the mass splitting Δm cannot be much larger than 125 GeV.

5 Conclusions

We have shown that it is quite natural in the triplet model for the lightest Higgs boson to have mass as large as 1 TeV. Although quantum corrections could play an important role in pushing up the Higgs mass we have shown that it is perhaps most natural to do this through tree-level corrections which arise due to mixing in the charged Higgs sector.

Appendix

Here we give a few details on the calculation of S, T and U in the triplet model. Starting from their definitions in eq. (16) we can write them in terms of the standard functions, A and B_{22} (up to order β^2 corrections):

$$
\begin{aligned}
S_{TM} &= 0, \qquad (27)\\
T_{TM} &= \frac{1}{4\pi}\frac{1}{s_W^2 c_W^2 m_Z^2}\left(4B_{22}(0;m_c,m_k) - A(m_c) - A(m_k)\right),\\
U_{TM} &= \frac{4}{\pi}\left(\frac{B_{22}(m_W^2;m_c,m_k) - B_{22}(0;m_c,m_k)}{m_W^2} - \frac{B_{22}(m_Z^2;m_c,m_c) - B_{22}(0;m_c,m_c)}{m_Z^2}\right)
\end{aligned}
$$

where

$$
\begin{aligned}
\frac{i}{(4\pi)^2}\, A(m) &= \mu^{4-D}\int \frac{d^D k}{(2\pi)^D}\frac{1}{k^2 - m^2 + i\epsilon}, \qquad (28)\\
\frac{i}{(4\pi)^2}\, g^{\mu\nu}\, B_{22}(p^2;m_1,m_2) &= \mu^{4-D}\int \frac{d^D k}{(2\pi)^D}\frac{k^\mu k^\nu}{(k^2 - m_1^2 + i\epsilon)((k-p)^2 - m_2^2 + i\epsilon)}. \qquad (29)
\end{aligned}
$$

These can be evaluated using dimensional regularisation [7], e.g.

$$
\begin{aligned}
A(m) &= m^2\left(\frac{1}{\epsilon} - \gamma_E + 1 - \log\left(\frac{m^2}{4\pi\mu^2}\right)\right), \qquad (30)\\
B_{22}(0;m_1,m_2) &= \frac{1}{4}\left[\left(\frac{1}{\epsilon} - \gamma_E + \frac{3}{2}\right)(m_1^2 + m_2^2)\right.\\
&\quad \left. - \frac{1}{m_1^2 - m_2^2}\left(m_1^4 \log\left(\frac{m_1^2}{4\pi\mu^2}\right) - m_2^4\log\left(\frac{m_2^2}{4\pi\mu^2}\right)\right)\right]. \qquad (31)
\end{aligned}
$$

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References

[1] LEP Electroweak Working Group, CERN-EP/2001-021 and references therein.

[2] D.C. Kennedy and B.W. Lynn, Nucl. Phys. B322 (1989) 1.

[3] M.E. Peskin and T. Takeuchi, Phys. Rev. Lett. 65 (1990) 964; Phys. Rev. D46 (1992) 381.

[4] H.M. Georgi, S.L. Glashow and S. Nussinov, Nucl. Phys. B193 (1981) 297; G. Passarino, Phys. Lett. B231 (1989) 458; Phys. Lett. B247 (1990) 587; B.W. Lynn and E. Nardi, Nucl. Phys. B381 (1992) 467; J.F. Gunion, R. Vega and J. Wudka, Phys. Rev. D42 (1990) 1673.

[5] T. Blank and W. Hollik, Nucl. Phys. B514 (1998) 113.

[6] D. Bardin et al, Comput. Phys. Commun. 133 (2001) 229.

[7] G. Passarino and M. Veltman, Nucl. Phys. B160 (1979) 151.